EURÊKA : Calcul Littéral
Corrigés

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CL01

a) $\frac{3x^2}{2y}$ b) $-3x$ c) $\frac{5x-4}{xy}$ d) $\frac{2}{8y-1}$

e) $\frac{7x-1}{x}$ f) $\frac{2x-3}{3}$ g) x h) $\frac{-1}{3x+1}$

i) $\frac{x}{3y}$ j) -1 k) $\frac{2x-3}{2x+4}$ l) $4xy$

CL02

a) $\frac{x^2-2xy+y^2}{3x-3y} = \frac{(x-y)^2}{3(x-y)} = \frac{x-y}{3}$

b) $\frac{x^2-2x-3}{x^2+x} = \frac{(x-3)\cdot(x+1)}{x\cdot(x+1)} = \frac{x-3}{x}$

c) $\frac{4x^2-9}{3-2x} = \frac{(2x-3)\cdot(2x+3)}{-2x+3} = \frac{-(2x-3)\cdot(2x+3)}{2x-3} = -2x-3$

d) $\frac{x^2+11x+30}{x^2-25} = \frac{(x+5)\cdot(x+6)}{(x+5)\cdot(x-5)} = \frac{x+6}{x-5}$

e) $\frac{9x^3-12x^2+4x}{3x^3-2x^2} = \frac{x\cdot(9x^2-12x+4)}{x\cdot(3x^2-2x)} = \frac{(3x-2)(3x-2)}{x(3x-2)} = \frac{3x-2}{x}$

f) $\frac{6x^4-96}{3x+6} = \frac{6\cdot(x^4-16)}{3\cdot(x+2)} = \frac{6\cdot(x^2-4)\cdot(x^2+4)}{3\cdot(x+2)} = \frac{6\cdot(x-2)\cdot(x+2)\cdot(x^2+4)}{3\cdot(x+2)} = 2\cdot(x-2)\cdot(x^2+4)$

g) $\frac{3x^2-30x+72}{-6x^2-12x+144} = \frac{3\cdot(x^2-10x+24)}{-6(x^2+2x-24)} = \frac{3\cdot(x-6)\cdot(x-4)}{-6\cdot(x+6)\cdot(x-4)} = \frac{x-6}{-2\cdot(x+6)}$

h) $\frac{9x^3-4x}{18x^2-24x+8} = \frac{x\cdot(9x^2-4)}{2\cdot(9x^2-12x+4)} = \frac{x\cdot(3x-2)\cdot(3x+2)}{2(3x-2)\cdot(3x-2)} = \frac{x\cdot(3x+2)}{2(3x-2)}$

i) $\frac{x^2-2x-48}{x^2+14x+48} = \frac{(x-8)\cdot(x+6)}{(x+8)\cdot(x+6)} = \frac{x-8}{x+8}$

j) $\frac{2-2x}{6x^2-6} = \frac{2\cdot(1-x)}{6\cdot(x^2-1)} = \frac{1-x}{3\cdot(x-1)\cdot(x+1)} = \frac{x-1}{-3\cdot(x-1)(x+1)} = \frac{1}{-3\cdot(x+1)}$

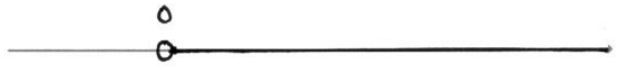
k) $\frac{75x^2+60x+12}{30x+12} = \frac{(5x+2)\cdot(15x+6)}{2\cdot(15x+6)} = \frac{5x+2}{2}$

l) $\frac{10x^2+90x-360}{5x^2+55x-60} = \frac{10\cdot(x^2+9x-36)}{5\cdot(x^2+11x-12)} = \frac{2\cdot(x+12)\cdot(x-3)}{(x+12)\cdot(x-1)} = \frac{2\cdot(x-3)}{x-1}$

CL03

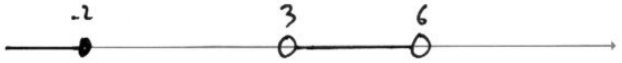
a. $]-\infty;3]$

b. $\mathbb{R}_+^*$

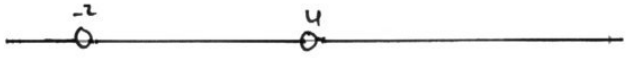


c. $]-\infty;-2]\cup]2;\infty[$

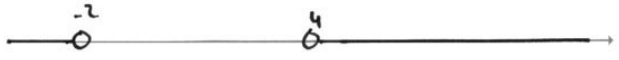
d. $]-\infty;-2]\cup]3;6[$



e. $\mathbb{R}\setminus\{-2;4\}$



f. $\mathbb{R}\setminus[-2;4]$



g. $\mathbb{R}\setminus]-2;4[$

h. $[-2;0]\cup[3;6[\cup]7;\infty[$

CL05

$ED_f = \mathbb{R}^*$ $ED_g = \mathbb{R} \setminus \{6\}$ $ED_h = \mathbb{R} \setminus \{2\}$

$ED_i = \mathbb{R}^* \setminus \{-1\}$ $ED_j = \mathbb{R} \setminus \{-34; 2; \pi\}$

CL06

$ED_f = [0; \infty[= \mathbb{R}_+$ $ED_g = [0; \infty[= \mathbb{R}_+$

$ED_h = [11; \infty[$ $ED_i =]-\infty; 0] = \mathbb{R}_-$

$ED_j = [1; \infty[$ $ED_k = \mathbb{R}$

CL07

$ED_f =]0; \infty[= \mathbb{R}_+^*$ $ED_g =]5; \infty[$

$ED_h = [-1; 0[\cup]0; \infty[$ $ED_i = [2; \infty[$

$ED_j =]-1; \infty[$ $ED_k =]-\infty; 2[$

CL11

c : nombre de casoars

σ : nombre d'ornithorynques

b : nombre de boas

pattes / masse / animaux
$$\begin{cases} c\cdot 2+\sigma\cdot 4=190 & (1)\\ c\cdot 75+\sigma\cdot 2+b\cdot 3.5=1677 & (2)\\ c+\sigma+b=66 & (3)\end{cases}$$

$$\begin{matrix}(1)\\ (2)\cdot 2\\ (3)\cdot 4\end{matrix}\begin{cases} 2c+4\sigma=190\\ 150c+4\sigma+7b=3354 & (4)\\ 4c+4\sigma+4b=264 & (5)\end{cases}$$

$$\begin{matrix}(4)-(1)\\ (4)-(5)\end{matrix}\begin{cases} 148c+7b=3164 & (6)\\ 146c+3b=3090 & (7)\end{cases}$$

$$\begin{matrix}(6)\cdot 3\\ (7)\cdot 7\end{matrix}\begin{cases} 444c+21b=9492 & (8)\\ 1022c+21b=21630 & (9)\end{cases}$$

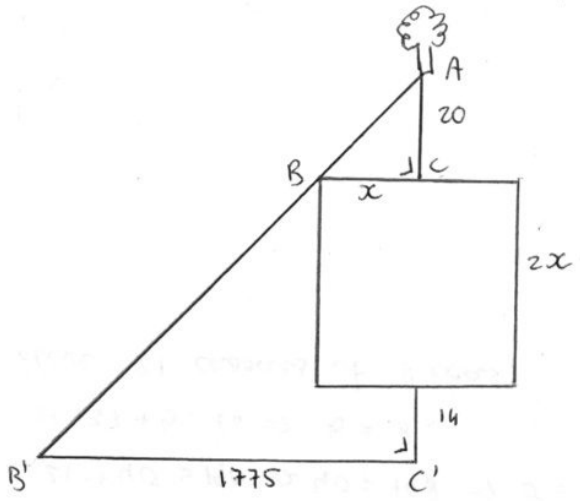
$(9)-(8)\quad 578c=12138 \Rightarrow c=\dfrac{12138}{578}=21$

$(1)\quad 2\cdot 21+4\sigma=190 \Rightarrow 4\sigma=148 \Rightarrow \sigma=37$

$(3)\quad 21+37+b=66 \Rightarrow b=8$

Elle élève 21 casoars et 8 boas.

CL12



Les triangles ABC et AB'C' sont semblables

$\Rightarrow \frac{BC}{AC} = \frac{B'C'}{AC'}$

$\frac{x}{20} = \frac{1775}{34+2x} \Rightarrow x(34+2x) = 20 \cdot 1775$

$2x^2 + 34x - 35\,500 = 0$

$a = 2$

$b = 34$

$c = -35000$

discriminant $\Delta = b^2 - 4ac = 34^2 - 4 \cdot 2 \cdot (-35\,500)$

$= 285\,156$

zéros : $\frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-34 \pm 534}{4} \Rightarrow 125$ et -142

x doit être positif $\Rightarrow x = 125$

$\Rightarrow$ les dimensions de la ville sont $2x$ par $2x$

$= \underline{250 \times 250}$ Li

CL18

a) MN : pente $= \frac{20-(-5)}{2-(-3)} = \frac{25}{5} = 5$

ordonnée à l'origine : $y = ax + b = 5x + b$

$20 = 5 \cdot 2 + b$

$b = 10$

$y = 5x + 10$

RS : pente $= \frac{1-4}{8-(-4)} = \frac{-3}{12} = -\frac{1}{4}$

ordonnée à l'origine : $y = -\frac{1}{4}x + b$

$4 = -\frac{1}{4} \cdot -4 + b$

$4 = 1 + b$

$b = 3$

$y = -\frac{1}{4}x + 3$

Intersection :

$5x + 10 = -\frac{1}{4}x + 3$

$5x + \frac{1}{4}x = 3 - 10$

$\frac{21}{4}x = -7$

$x = \frac{-7 \cdot 4}{21} = -\frac{28}{21} = \frac{-4}{3}$

$y = 5 \cdot \frac{-4}{3} + 10 = \frac{-20}{3} + 10 = \frac{-20}{3} + \frac{30}{3} = \frac{10}{3}$

$I = \left(-\frac{4}{3}; \frac{10}{3}\right)$

b) $y = -\frac{1}{4} \cdot \frac{5}{4} + 3 = -\frac{5}{16} + \frac{48}{16} = \frac{43}{16}$; T $\left(\frac{5}{4} ; \frac{43}{16}\right)$

c) $\frac{5}{9} = 5x + 10$

$5x = \frac{5}{9} - 10 = \frac{5}{9} - \frac{90}{9} = \frac{-85}{9}$

$x = \frac{-85}{45} = \frac{-17}{9}$ P $\left(\frac{-17}{9} ; \frac{5}{9}\right)$

CL19

pente $= \frac{-1-5}{3-(-2)} = \frac{-6}{5}$

$-1 = -\frac{6}{5} 3 + b$

$-1 = \frac{-18}{5} + b$

$b = -1 + \frac{18}{5} = \frac{-5}{5} + \frac{18}{5} = \frac{13}{5}$

$y = -\frac{6}{5} x + \frac{13}{5}$

c: $y = -\frac{6}{5} \cdot 7 + \frac{13}{5} = \frac{7}{5}$

CL20

a) $-8 = -3 \cdot 3 + b$

$b = 1$

$y = -3x + 1$

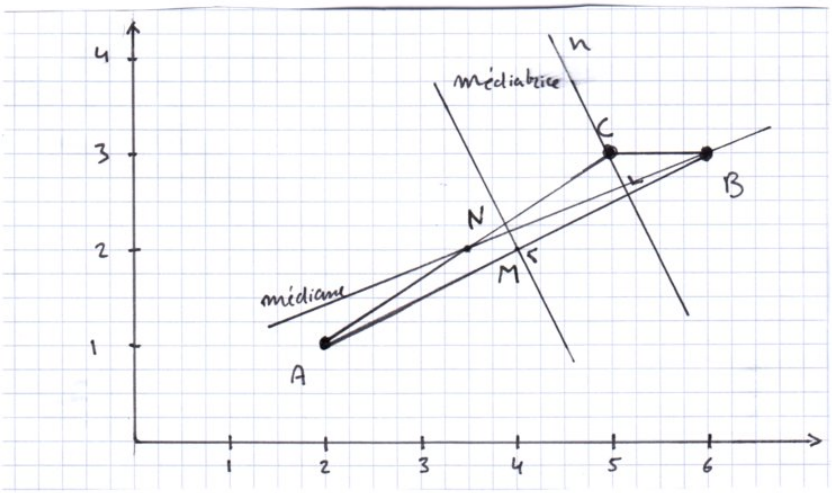
b) $a \cdot -3 = -1 \Rightarrow a = \frac{1}{3}$

$-8 = \frac{1}{3} \cdot 3 + b = 1 + b$

$b = -9$

$y = \frac{1}{3} x - 9$

CL21



droite AB:

pente $= \frac{3-1}{6-2} = \frac{2}{4} = \frac{1}{2}$

$\frac{1}{2} \cdot 2 + b = 1 \Rightarrow b = 0$

$y = \frac{1}{2}x$

a) pente de la hauteur: $\frac{1}{2}a = -1 \Rightarrow a = -2$

$-2 \cdot 5 + b = 3$

$-10 + b = 3$

$b = 13$

hauteur: $y = -2x + 13$

b) $M\left(\frac{6+2}{2}; \frac{1+3}{2}\right) = (4; 2)$

pente de la médiatrice = pente de la hauteur $= -2$

$-2 \cdot 4 + b = 2$

$-8 + b = 2$

$b = 10$

médiatrice: $y = -2x + 10$

c) $N\left(\frac{2+5}{2}\,;\,\frac{1+3}{2}\right) = (3.5\,;\,2)$

pente de la médiane : $\frac{3-2}{6-3.5} = \frac{1}{2.5} = \frac{2}{5}$

$\frac{2}{5}\cdot 6 + b = 3$

$\frac{12}{5} + b = 3$

$b = 3 - \frac{12}{5} = \frac{3}{5}$

$\Rightarrow y = \frac{2}{5}x + \frac{3}{5}$

CL22

a) pente de p : $2\cdot a = -1 \Rightarrow a = -\frac{1}{2}$

$-\frac{1}{2}\cdot 8 + b = 11$

$-4 + b = 11$

$b = 15 \quad \Rightarrow y = -\frac{1}{2}x + 15$

b) $2x + 6 = -\frac{1}{2}x + 15$

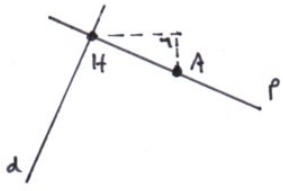
$\frac{4}{2}x + 6 = -\frac{1}{2}x + 15$

$\frac{5}{2}x = 9 \Rightarrow 5x = 18 \Rightarrow x = \frac{18}{5}$

$y = 2\cdot\frac{18}{5} + 6$

$y = \frac{36}{5} + 6 \Rightarrow y = \frac{66}{5} \Rightarrow H\left(\frac{18}{5}\,;\,\frac{66}{5}\right)$

c)



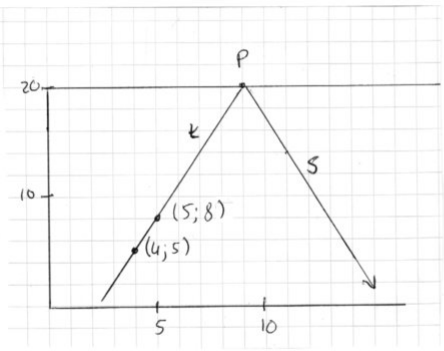
Pythagore :

$d = \sqrt{\left(8 - \frac{18}{5}\right)^2 + \left(\frac{66}{5} - 11\right)^2} =$

$\sqrt{\left(\frac{40}{5} - \frac{18}{5}\right)^2 + \left(\frac{66}{5} - \frac{55}{5}\right)^2} =$

$\sqrt{\left(\frac{22}{5}\right)^2 + \left(\frac{11}{5}\right)^2} = \sqrt{\frac{605}{25}} = \frac{11\sqrt{5}}{5}$

CL23



Equation de t :

$\begin{cases} 5 = a \cdot 4 + b \\ 8 = a \cdot 5 + b \end{cases}$ $\begin{cases} 4a + b = 5 \quad (1) \\ 5a + b = 8 \quad (2) \end{cases}$

(2) − (1) : $a = 3$

(1) $4 \cdot 3 + b = 5 \Rightarrow b = -7$

$y = 3x - 7$

Passe-t-elle par A :

$y = 3 \cdot 8 - 7 = \underline{17}$ oui

coordonnées de P :

$P \in t \Rightarrow 20 = 3x - 7 \Rightarrow 3x = 27 \Rightarrow x = 9$

$P(9;20)$

équation de s $(P \in s)$

pente $= -3$

$\Rightarrow y = -3x + b$

passe par P $\Rightarrow 20 = -3 \cdot 9 + b \Rightarrow b = 47$

$y = -3x + 47$

Passe-t-elle par B :

$y = -3 \cdot 15 + 47 = 2 \Rightarrow \underline{\text{non}}$

Passe-t-elle par C :

$y = -3 \cdot 37 + 47 = -64$ oui

CL29

a) $4 = a \cdot 0 + b \Rightarrow b = 4 \Rightarrow f_m : x \mapsto mx + 4$

b) $5 = a \cdot 2 + b \Rightarrow b = 5 - 2a = -2a + 5$

$g_a : x \mapsto ax - 2a + 5$

$g_m : x \mapsto mx - 2m + 5$

c) $3 = a \cdot -2 + b$

$b = 2a + 3$

$h_a : x \mapsto ax + 2a + 3$

$h_m : x \mapsto mx + 2m + 3$

CL30

a) $m = 5$

b) $3m - 1 = m \Rightarrow m = \frac{1}{2}$

c) $(2m+3)^2 = 4m(m+3)$

$4m^2 + 9 + 12m = 4m^2 + 12m$

$9 = 0 \Rightarrow \phi$

d) $5m - 1 = m^2 - 25$

$m^2 - 5m - 24 = 0$

$(m-8)(m+3) = 0 \qquad m \in \{-3 ; 8\}$

e) $10m - 72 = 2m(m+20)$

$10m - 72 = 2m^2 + 40m$

$2m^2 + 30m + 72 = 0$

$2(m^2 + 15m + 36) = 0$

$2(m+12)\cdot(m+3) = 0 \qquad m \in \{-12 ; -3\}$

f) $9m^2 - 6m = -1$

$9m^2 - 6m + 1 = 0$

$9\left(m^2 - \frac{2}{3}m + \frac{1}{9}\right) = 0$

$9\left(m - \frac{1}{3}\right)^2 = 0 \quad ; \quad m = \frac{1}{3}$

CL31

a) $m = -1$

b) $m + 2 = m + 4 \Rightarrow 2 = 4 \Rightarrow \phi$

c) $5m^2 - 10m + 20 = m^2 + 10m - 5$

$4m^2 - 20m + 25 = 0$

$(2m-5)^2 = 0 \Rightarrow m = \frac{5}{2}$

d) $m^2 - 2m = -2m + 36$

$m^2 = 36 \Rightarrow m = \pm 6 \qquad m \in \{-6 ; 6\}$

CL32

1. a) $m = 5$ b) ϕ c) $m \neq 5$ d) $m = -\frac{1}{5}$

2. a) jamais b) $m = 5$ c) $m \neq 5$ d) $m = -\frac{1}{5}$

3. a) $m = 2$ b) jamais c) $m \neq 2$ d) $m = -\frac{1}{50}$

4. a) $(m+1)^2 \neq 25 \Rightarrow (m+1) \neq \pm 5 \Rightarrow m \notin \{-6; 4\}$
   b) $m \in \{-6; 4\}$
   c) jamais
   d) jamais

5. même pente : $9 = (m-1)^2 \Rightarrow m - 1 = \pm 3 \Rightarrow m = 4$ ou $m = -2$
   même ordonnée à l'origine :
   $18 = 2 \cdot (m-1)^2 \Rightarrow 9 = (m-1)^2 \Rightarrow m = 4$ ou $m = -2$
   a) jamais (si même pente $\Rightarrow$ même ordonnée à l'origine)
   b) $m \in \{-2; 4\}$
   c) $m \notin \{-2; 4\}$
   d) jamais $(m-1)^2 = -\frac{1}{9}$, impossible dans $\mathbb{R}$

6. même pente : $2m = 20$; $m = 10$
   même ordonnée à l'origine : $\frac{m}{2} = 4 \Rightarrow m = 8$
   a) $m = 10$
   b) jamais ($m = 10$ et $m = 8$)
   c) $m \neq 10$
   d) $2m = -\frac{1}{20}$; $m = -\frac{1}{40}$

7. même pente : $m + 6 = 6 \Rightarrow m = 0$
   même ordonnée à l'origine : $m = 1$
   a) $m = 0$
   b) jamais
   c) $m \neq 0$
   d) $m + 6 = -\frac{1}{6}$
   $m + \frac{36}{6} = -\frac{1}{6} \Rightarrow m = -\frac{37}{6}$

CL33

1. Même pente : $2m = 3$; $m = \frac{3}{2}$
   même ordonnée à l'origine $m = -5$
   a) $m = \frac{3}{2}$
   b) jamais
   c) $m \neq \frac{3}{2}$

2. même pente : $m - 1 = 2m \Rightarrow m = -1$
   même ordonnée à l'origine : $m = -1$
   a) jamais (si même pente elles sont confondues)
   b) $m = -1$
   c $m \neq -1$

3. même pente :
   $m^2 = 3m$
   $m^2 - 3m = 0$
   $m(m-3) = 0$ $m = 0$ ou 3
   même ordonnée à l'origine : $-2m = 0$; $m = 0$
   a) $m = 3$
   b) $m = 0$
   c) $m \notin \{0; 3\}$

4. même pente : $1 = 2 \Rightarrow$ jamais
   même ordonnée à l'origine : $m = -1$
   a) jamais
   b) jamais
   c) toujours

5. a) jamais
   b) jamais
   c) toujours en $(0; m)$

6. même pente :

$2(m^2-1)=0$

$2m^2-2=0$

$2m^2=2$

$m^2=1$ $\quad m \in \{-1;1\}$

même ordonnée à l'origine :

$0=m+1$; $m=-1$

a) $m=1$

b) $m=-1$

c) $m \notin \{-1;1\}$

7. même pente : $m \in \mathbb{R}$

même ordonnée à l'origine :

$3m=m(m-1)$

$m^2-4m=0$

$m(m-4)=0$ $\quad m \in \{0;4\}$

a) $m \notin \{0;4\}$

b) $m \in \{0;4\}$

c) jamais

8. même pente :

$m\cdot(m-4)=m^2-16$

$m^2-4m=m^2-16$

$4m=16$

$m=4$

même ordonnée à l'origine :

$2m=-m-5$

$3m=-5$

$m=-\frac{5}{3}$

a) $m=4$

b) jamais

c) $m \neq 4$

3. même pente :

$5-m = m+2$

$5-2m = 2$

$-2m = -3$

$m = \frac{3}{2}$

même ordonnée à l'origine :

$-2m-3 = -6m$

$4m = 3$

$m = \frac{3}{4}$

a) $m = \frac{3}{2}$

b) jamais

c) $m \neq \frac{3}{2}$

CL34

a) $5m-2(3m-2x) = m-2x$

$5m-6m+4x = m-2x$

$-m+4x = m-2x \quad | +m+2x$

$6x = 2m \quad | :6$

$x = \frac{m}{3}$ $\qquad S = \left\{\frac{m}{3}\right\}$

b) $3+2(m-7x) = 4(m-3x)-2(m+x)+3$

$3+2m-14x = 4m-12x-2m-2x+3$

$3+2m-14x = 2m-14x+3 \quad | -2m+14x$

$3 = 3 \Rightarrow S = \mathbb{R}$

c) $mx+2m-8 = m(x+2)+3$

$mx+2m-8 = mx+2m+3 \quad | -mx-2m$

$-8 = 3 \Rightarrow S = \emptyset$

d) $mx+(m+2)(3x-1)=3(2x+1)$

$mx+3mx-m+6x-2=6x+3 \quad |-6x$

$4mx-m-2=3 \quad |+m$

$4mx-2=m+3 \quad |+2$

$4mx=m+5 \quad |:4m$

$x=\frac{m+5}{4m} \Rightarrow$ Si $m=0$ $S=\emptyset$

sinon $S=\left\{\frac{m+5}{4m}\right\}$

e) $2mx-3(m-1)=m+3$

$2mx-3m+3=m+3 \quad |-m-3$

$2mx-4m=0 \quad |+4m$

$2mx=4m$

$mx=2m$

Si $m=0 \Rightarrow 0x=0 \Rightarrow S=\mathbb{R}$

Si $m\neq 0 \Rightarrow x=2 \Rightarrow S=\{2\}$

f) $4x(m+3)=3x+7m$

$4mx+12x=3x+7m \quad |-3x-7m$

$4mx+9x-7m=0 \quad |+7m$

$(4m+9)\cdot x=7m$

Si $m=-\frac{9}{4} \Rightarrow 0x=7\cdot\frac{-9}{4}=\frac{-63}{4} \Rightarrow S=\emptyset$

Si $m\neq\frac{-9}{4} \Rightarrow \frac{(4m+9)x}{4m+9}=\frac{7m}{4m+9}$

$x=\frac{7m}{4m+9} \Rightarrow S=\left\{\frac{7m}{4m+9}\right\}$

g) $(m-5)(2x+3)=5m+2x-3$

$2mx+3m-10x-15=5m+2x-3 \quad |-2x-3m+15$

$2mx-12x=2m+12$

$2(m-6)x=2(m+6) \quad |:2$

$(m-6)x=m+6$

Si $m=6 \Rightarrow 0x=12 \Rightarrow S=\emptyset$

Si $m\neq 6 \Rightarrow x=\frac{m+6}{m-6}$; $S=\left\{\frac{m+6}{m-6}\right\}$

h) $m-2(3m-x)=m(3-4x)$
$m-6m+2x=3m-4mx \quad |+5m+4mx$
$2x+4mx=8m$
$2(1+2m)x=8m \quad |:2$
$(1+2m)x=4m$
Si $1+2m=0 \Rightarrow m=-\frac{1}{2} \Rightarrow 0x=4\cdot\frac{-1}{2}=-2 \Rightarrow S=\emptyset$
si $m\neq-\frac{1}{2} \Rightarrow x=\frac{4m}{2m+1} \Rightarrow S=\left\{\frac{4m}{2m+1}\right\}$

CL35

a) $m(m-2)x=m^2$
Si $m=0 \Rightarrow 0x=0 \Rightarrow S=\mathbb{R}$
Si $m=2 \Rightarrow 0x=4 \Rightarrow S=\emptyset$
Si $m\notin\{0;2\}$:
$x=\frac{m^2}{m(m-2)} \Rightarrow S=\left\{\frac{m^2}{m(m-2)}\right\}$

b) $m(m+1)x+1=m^2$
$m(m+1)x=m^2-1$
Si $m=0 \Rightarrow 0=-1 \Rightarrow S=\emptyset$
Si $m=-1 \Rightarrow 0x=0 \Rightarrow S=\mathbb{R}$
Si $m\notin\{-1;0\}$:
$x=\frac{m^2-1}{m(m+1)}=\frac{(m-1)(m+1)}{m(m+1)}=\frac{m-1}{m} \Rightarrow S=\left\{\frac{m-1}{m}\right\}$

c) $m(mx-1)=9x+3$
$m^2x-m=9x+3 \quad |-9x+m$
$m^2x-9x=m+3$
$(m^2-9)x=m+3$
$(m-3)(m+3)x=m+3$
Si $m=3 \Rightarrow 0x=6 \Rightarrow S=\emptyset$
Si $m=-3 \Rightarrow 0x=0 \Rightarrow S=\mathbb{R}$
Si $m\notin\{-3;3\}$:
$x=\frac{m+3}{(m-3)(m+3)}=\frac{1}{m-3} \quad S=\left\{\frac{1}{m-3}\right\}$

d) $m(mx+4) = 3m^2$

$m^2x + 4m = 3m^2 \mid -4m$

$m^2x = 3m^2 - 4m$

Si $m = 0 \Rightarrow 0x = 0 \Rightarrow S = \mathbb{R}$

Si $m \neq 0$:

$x = \frac{3m^2-4m}{m^2} = \frac{m(3m-4)}{m^2} = \frac{3m-4}{m}$

$\Rightarrow S = \left\{ \frac{3m-4}{m} \right\}$

e) $3mx(3m-10) = 3m - 5(5x+1)$

$9m^2x - 30mx = 3m - 25x - 5 \mid +25x$

$9m^2x - 30mx + 25x = 3m - 5$

$(9m^2 - 30m + 25)x = 3m - 5$

$(3m-5)(3m-5)x = 3m - 5$

Si $m = \frac{5}{3} \Rightarrow 0x = 3\frac{5}{3} - 5 = 0 \Rightarrow S = \mathbb{R}$

Si $m \neq \frac{5}{3}$:

$x = \frac{3m-5}{(3m-5)(3m-5)} = \frac{1}{3m-5} \Rightarrow S = \left\{ \frac{1}{3m-5} \right\}$

f) $m^2(x-1) + m(7x-4) = 4 - 10x$

$m^2x - m^2 + 7mx - 4m = 4 - 10x \mid +10x + m^2 + 4m$

$m^2x + 7mx + 10x = m^2 + 4m + 4$

$(m+2)(m+5)x = (m+2)(m+2)$

si $m = -5$: $0x = -3 \cdot -3 = 9 \Rightarrow S = \emptyset$

si $m = -2$ $0x = 0 \Rightarrow S = \mathbb{R}$

si $m \notin \{-5; -2\} \Rightarrow x = \frac{(m+2)(m+2)}{(m+2)(m+5)} = \frac{m+2}{m+5}$

$\Rightarrow S = \left\{ \frac{m+2}{m+5} \right\}$

g) $4m(x+1) = 2 + x(4m^2+1)$

$4mx + 4m = 2 + 4m^2x + x \quad | -4m^2x - x - 4m$

$-4m^2x + 4mx - x = -4m + 2$

$x(-4m^2 + 4m - 1) = -4m + 2$

$-x(4m^2 - 4m + 1) = -4m + 2$

$-x(2m-1)^2 = -2(2m-1)$

Si $m = \frac{1}{2} \Rightarrow 0x = -2(2\frac{1}{2} - 1) = 0 \Rightarrow S = \mathbb{R}$

Si $m \neq \frac{1}{2}$:

$-x = \frac{-2(2m-1)}{(2m-1)^2} = \frac{-2}{2m-1} \Rightarrow x = \frac{2}{2m-1}$

$\Rightarrow S = \left\{ \frac{2}{2m-1} \right\}$

h) $mx(m-4) - m(m-10) = 12(x+2)$

$m^2x - 4mx - m^2 + 10m = 12x + 24 \quad | +m^2 - 10m - 12x$

$m^2x - 4mx - 12x = m^2 - 10m + 24$

$(m^2 - 4m - 12)x = m^2 - 10m + 24$

$(m+2)(m-6)x = (m-6)(m-4)$

Si $m = 6 \Rightarrow 0 = 0 \Rightarrow S = \mathbb{R}$

Si $m = -2 \Rightarrow 0 = -8 \cdot -6 = 48 \Rightarrow S = \phi$

Si $m \notin \{-2; 6\}$:

$x = \frac{(m-6)(m-4)}{(m+2)(m-6)} = \frac{m-4}{m+2} \Rightarrow S = \left\{ \frac{m-4}{m+2} \right\}$

CL36

1) $3x - 2(x+m) = 4x - 6m$

$x - 2m = 4x - 6m \quad | -x + 6m$

$4m = 3x$

$x = \frac{4}{3}m$

a) impossible

b) impossible

c) $8 = \frac{4}{3}m \Rightarrow m = \frac{24}{4} = 6$

d) $-1 = \frac{4}{3}m \Rightarrow m = \frac{-3}{4}$

2) $4x - m = m(x-2) + 7$

$4x - m = mx - 2m + 7 \quad | -mx + m$

$4x - mx = -m + 7$

$(4-m)x = -m + 7$

a) si $m = 4 \Rightarrow \quad 0 = 3 \Rightarrow$ impossible

b) si $m = 4 \Rightarrow 0 = 3 \Rightarrow m = 4$

c) $(4-m)8 = -m + 7$

$32 - 8m = -m + 7 \quad | + m - 32$

$-7m = -25$

$m = \frac{25}{7}$

d) $(4-m)(-1) = -m + 7 \; ; \; -4 + m = -m + 7$

$-4 + 2m = 7 \Rightarrow m = \frac{11}{2}$

3) $mx + (m+1)(x+1) = 6m(m+1) + 2$

$mx + mx + m + x + 1 = 6m^2 + 6m + 2 \quad | -m - 1$

$2mx + x = 6m^2 + 5m + 1$

$(2m+1)x = (2m+1)(3m+1) \; ; \; x = 3m + 1$

a) $2m + 1 = 0 \Rightarrow m = -\frac{1}{2}$

$0x = \left(2 \cdot -\frac{1}{2} + 1\right)\left(3 \cdot -\frac{1}{2} + 1\right) = 0 \Rightarrow S = \mathbb{R}$ si $m = -\frac{1}{2}$

b) impossible

c) $(2m+1)8 = 6m^2 + 5m + 1$

$16m + 8 = 6m^2 + 5m + 1 \quad | -16m - 8$

$6m^2 - 11m - 7 = 0$

$\left(m - \frac{7}{3}\right)\left(m + \frac{1}{2}\right) = 0$

si $m = -\frac{1}{2} \Rightarrow S = \mathbb{R}$ (comme en a)

Si $m = \frac{7}{3} \Rightarrow S = \{8\}$

d) $(2m+1)(-1) = 6m^2 + 5m + 1$

$-2m - 1 = 6m^2 + 5m + 1 \quad | +2m + 1$

$6m^2 + 7m + 2 = 0$

$\left(m + \frac{2}{3}\right)\left(m + \frac{1}{2}\right) = 0$

Si $m = -\frac{1}{2}$, $S = \mathbb{R}$

si $m = -\frac{2}{3} \Rightarrow S = \{-1\}$

4) $EV = \mathbb{R} \setminus \{-1; 1\}$

$$\frac{x}{m-1} + \frac{x}{m+1} = \frac{1}{m+1}$$

$$\frac{x(m+1) + x(m-1)}{(m-1)(m+1)} = \frac{1}{(m+1)}$$

$$\frac{x(m+1) + x(m-1)}{m-1} = 1 \quad | \cdot (m-1)$$

$$x(m+1) + x(m-1) = m-1$$

$$mx + x + mx - x = m-1$$

$$2mx = m-1$$

a) $2m = 0 \Rightarrow m = -1 \Rightarrow$ impossible $(0 = -1)$

b) si $m = 0$

c) $2m \cdot 8 = m-1$

$16m = m-1 \quad | -m$

$15m = -1$

$m = -\frac{1}{15}$ si $m = -\frac{1}{15}$, $S = \{8\}$

d) $2m(-1) = m-1$

$-2m = m-1 \quad | -m$

$-3m = -1$

$m = \frac{1}{3}$ si $m = \frac{1}{3}$, $S = \{-1\}$

5) $EV = \mathbb{R}^*$

$$\frac{1}{m} + x = m - \frac{x}{m} \quad | \cdot m$$

$$m\left(\frac{1}{m} + x\right) = m\left(m - \frac{x}{m}\right)$$

$$\frac{m}{m} + mx = m^2 - \frac{mx}{m}$$

$$1 + mx = m^2 - x \quad | +x - 1$$

$$mx + x = m^2 - 1$$

$$(m+1)x = m^2 - 1$$

a) si $m + 1 = 0 \Rightarrow m = -1$

$0x = 0 \Rightarrow S = \mathbb{R}$ si $m = -1$

b) jamais

c) $(m+1)8 = m^2 - 1$

$8m + 8 = m^2 - 1 \quad | -8m - 8$

$m^2 - 8m - 9 = 0$

$(m+1)(m-9) = 0$

Si $m = -1 \Rightarrow S = \mathbb{R}$ (voir a)

Si $m = 9 \Rightarrow S = \{8\}$

d) $(m+1)(-1) = m^2 - 1$

$-m - 1 = m^2 - 1 \quad | +m + 1$

$m^2 + m = 0$

$m(m+1) = 0$

Si $m = -1 \Rightarrow S = \mathbb{R}$ (voir a)

$m = 0 \notin$ E.V.

$\Rightarrow$ aucune valeur possible pour $S = \{-1\}$

CL37

a) $EV = \mathbb{R} \setminus \{3\}$

$\frac{x}{3} - \frac{2}{m-3} = 1 \quad | + \frac{2}{m-3}$

$\frac{x}{3} = 1 + \frac{2}{m-3} = \frac{m-3+2}{m-3} = \frac{m-1}{m-3}$

$x = \frac{3(m-1)}{m-3} = \frac{3m-3}{m-3} \Rightarrow S = \left\{\frac{3m-3}{m-3}\right\}$

b) $EV = \mathbb{R}^*$

$\frac{5m-3}{m^2} - \frac{x}{m} = \frac{4x}{m} \quad | + \frac{x}{m}$

$\frac{5m-3}{m^2} = \frac{5x}{m} \quad | \cdot m$

$\frac{5m-3}{m} = 5x \quad | : 5$

$x = \frac{5m-3}{5m} \qquad S = \left\{\frac{5m-3}{5m}\right\}$

c) $EV = \mathbb{R}^*$

$$\frac{x+2}{m} + \frac{2x-3}{3m} = \frac{m}{6}$$

$$\frac{3x+6+2x-3}{3m} = \frac{m}{6} \quad \Big| \cdot 3m$$

$$5x+3 = \frac{m^2}{2}$$

$$5x = \frac{m^2}{2} - 3 = \frac{m^2-6}{2} \quad \Big| :5$$

$$x = \frac{m^2-6}{10} \qquad S = \left\{ \frac{m^2-6}{10} \right\}$$

d) $EV : \mathbb{R} \setminus \{2\}$

$$\frac{2x}{m-2} + \frac{m}{2} = x \quad \Big| -x - \frac{m}{2}$$

$$\frac{2x}{m-2} - x = -\frac{m}{2}$$

$$\frac{2x - x(m-2)}{m-2} = \frac{-m}{2}$$

$$\frac{2x - mx + 2x}{m-2} = -\frac{m}{2} \quad \Big| \cdot (m-2)$$

$$4x - mx = \frac{-m(m-2)}{2}$$

$$x(4-m) = \frac{-m(m-2)}{2}$$

$$x = \frac{-m(m-2)}{2(4-m)} = \frac{m(m-2)}{-2(4-m)} = \frac{m(m-2)}{2(m-4)}$$

Si $m = 4$:

$$\frac{2x}{4-2} + \frac{4}{2} = x$$

$$x + 2 = x \Rightarrow S = \emptyset$$

Si $m \neq 4$ $\qquad S = \left\{ \frac{m(m-2)}{2(m-4)} \right\}$

e) $EV = \mathbb{R}^* \setminus \{-1\}$

$$\frac{5}{m} - \frac{x}{m+1} = 3 \quad \Big| -\frac{5}{m}$$

$$\frac{-x}{m+1} = 3 - \frac{5}{m} = \frac{3m-5}{m} \quad \Big| \cdot (m+1)$$

$$-x = \frac{(3m-5)(m+1)}{m} = \frac{3m^2-2m-5}{m} \quad \Big| \cdot -1$$

$$x = \frac{-3m^2+2m+5}{m}$$

$$S = \left\{ \frac{-3m^2+2m+5}{m} \right\}$$

f) $m^2 - 3m - 10 = (m-5)(m+2) \Rightarrow EV = \mathbb{R} \setminus \{-2; 5\}$

$$\frac{mx}{m+2} + \frac{(5m-4)x}{m^2-3m-10} = 1 = \frac{mx}{m+2} + \frac{(5m-4)x}{(m-5)(m+2)} =$$

$$\frac{mx(m-5)+(5m-4)x}{(m-5)(m+2)} = \frac{m^2x-5mx+5mx-4x}{(m-5)(m+2)} =$$

$$\frac{mx^2-4x}{(m-5)(m+2)} = 1$$

$$mx^2-4x = (m-5)(m+2)$$

$$x(m^2-4) = (m-5)(m+2)$$

$$x = \frac{(m-5)(m+2)}{(m-2)(m+2)} = \frac{m-5}{m-2}$$

Si $m = 2 \Rightarrow S = \emptyset$

si $m \neq 2 \Rightarrow S = \left\{ \frac{m-5}{m-2} \right\}$

g) $EV = \mathbb{R} \setminus \{3\}$

$$\frac{2m(x-6)}{2m-6} + \frac{x(9m+10)}{2} = \frac{2-14mx-17x}{m-3}$$

$$\frac{2m(x-6)+x(9m-10)(2m-6)}{2(2m-6)} = \frac{2-14mx-17x}{m-3}$$

$$\frac{2m(x-6)+x(9m-10)(2m-6)}{4(m-3)} = \frac{2-14mx-17x}{m-3}$$

$$2m(x-6)+x(9m-10)(2m-6) = 2-14mx-17x$$

$$2mx-12m+18mx^2-54mx+20mx-60x = 8-56mx-68x$$

$$-12m+18m^2x-32mx-60x = 8-56mx-68x \quad | +56mx+68x$$

$$-12m+18m^2x+24mx+8x = 8 \quad | +12m$$

$$(18m^2+24m+8)x = 12m+8$$

$$(9m^2+12m+4)2x = 12m+8$$

$$(9m^2+12m+4)x = 6m+4$$

$$(m+\tfrac{2}{3})^2 x = 6m+4$$

Si $m = -\frac{2}{3}$: $0x = 6\cdot\frac{-2}{3}+4 = 0 \Rightarrow S = \mathbb{R}$

si $m \neq -\frac{2}{3}$:

$$x = \frac{6m+4}{(m+\frac{2}{3})^2} = \frac{6m+4}{(3m+2)^2} = \frac{2(3m+2)}{(3m+2)^2} = \frac{2}{3m+2}$$

$$S = \left\{\frac{2}{3m+2}\right\}$$

h) $m^2-1 = (m-1)(m+1) \Rightarrow \mathbb{E}V = \mathbb{R} \setminus \{-1;1\}$

$$x+2 = \frac{(m+1)(x+2m-2)+4x}{m^2-1}$$

$$x+2 = \frac{mx+2m^2-2m+x+2m-2+4x}{m^2-1} \quad | \cdot (m^2-1)$$

$$(x+2)(m^2-1) = mx+2m^2+5x-2$$

$$m^2x-x+2m^2-2 = mx+2m^2+5x-2 \quad | -2m^2+2$$

$$m^2x-x = mx+5x \quad | -mx+5x$$

$$m^2x-mx-6x = 0$$

$$(m^2-m-6)x = 0$$

$$x(m-3)(m+2) = 0$$

Si $m=3 \Rightarrow 0x=0 \Rightarrow S=\mathbb{R}$

Si $m=-2 \Rightarrow 0x=0 \Rightarrow S=\mathbb{R}$

Si $m \notin \{-2;3\} \Rightarrow x=0 \Rightarrow S=\{0\}$

CL39

a) $\begin{cases} a\cdot 2^2+b\cdot 2+c=9 \\ a\cdot 3^6+b(-6)+c=-7 \\ a\cdot 1^2+b\cdot 1+c=0 \end{cases}$ $\begin{cases} 4a+2b+c=9 & (1) \\ 36a-6b+c=-7 & (2) \\ a+b+c=0 & (3) \end{cases}$

(1)-(3) $\begin{cases} 3a+b=9 & (4) \end{cases}$ (4)·7 $\begin{cases} 21a+7b=63 & (6) \end{cases}$

(2)-(3) $\begin{cases} 35a-7b=-7 & (5) \end{cases}$ (5) $\begin{cases} 35a-7b=-7 \end{cases}$

$f: x \to x^2+6x-7$

(6)-(5) $56a=56 \Rightarrow a=1$

(4) $3+b=9 \Rightarrow b=6$

(3) $1+6+c=0 \Rightarrow c=-7$

b) $\begin{cases} a\cdot 0^2+b\cdot 0+c=0 \\ a\cdot 3^2+b\cdot 3+c=-6 \\ a(-3)^2+b\cdot(-3)+c=12 \end{cases}$ $\begin{cases} c=0 \\ 9a+3b=-6 & (1) \\ 9a-3b=12 & (2) \end{cases}$

$g: x \to \frac{1}{3}x^2-3x$

(1)-(2) $6b=-18 \Rightarrow b=-3$

(1) $9a-9=-6 \Rightarrow a=\frac{1}{3}$

CL40

a) $S \in f \begin{cases} \frac{-b}{2a} = -1 \\ a\cdot(-1)^2 + b(-1) + c = -9 \\ a\cdot 3^2 + b\cdot 3 + c = 7 \end{cases} \Leftrightarrow \begin{cases} 2a - b = 0 & (1) \\ a - b + c = -9 & (2) \\ 9a + 3b + c = 7 & (3) \end{cases}$

$(3)-(2)$ $\begin{cases} 8a + 4b = 16 & (4) \\ 8a - 4b = 0 & (5) \end{cases}$ $(4)-(5)$ $8b = 16 \Rightarrow b = 2$

$(1)\cdot 4$ (1) $2a - 2 = 0 \Rightarrow a = 1$

(3) $9 + 6 + c = -7 \Rightarrow c = -8$

$$y = x^2 + 2x - 8$$

b)

1. $a = 1 ; b = -3 ; c = 2 ; \Delta = 3^2 - 4\cdot 1\cdot 2 = 1$

Zéros : $\frac{-b \pm \sqrt{\Delta}}{2a} = \frac{3 \pm 1}{2} \Rightarrow x_1 = 1 ; x_2 = 2$

sommet $= \left(\frac{-b}{2a} ; \frac{-\Delta}{4a}\right) = \left(\frac{3}{2} ; -\frac{1}{4}\right)$

ordonnée à l'origine : 2

2. $a = 1 ; b = 6 ; c = 9 ; \Delta = 6^2 - 4\cdot 1\cdot 9 = 0$

Zéros : $\frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-6 \pm \sqrt{0}}{2} \Rightarrow x_1 = x_2 = -3$

sommet : $\left(\frac{-b}{2a} ; \frac{-\Delta}{4a}\right) = \left(\frac{-6}{2} ; \frac{0}{4}\right) = (-3 ; 0)$

ordonnée à l'origine : 9

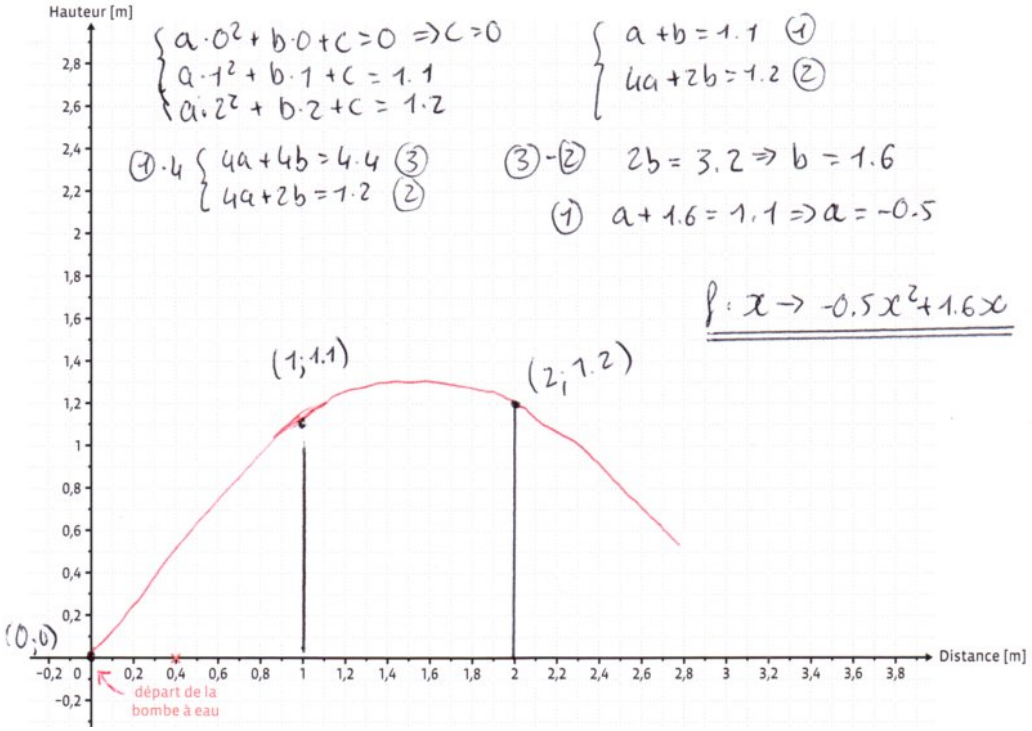
3. $a = 2 ; b = -1 ; c = 2 ; \Delta = 1 - 4\cdot 2\cdot 2 = -15$

$\Delta < 0 \Rightarrow$ pas de zéros

sommet $\left(\frac{-(-1)}{2\cdot 2} ; \frac{-(-15)}{4\cdot 2}\right) = \left(\frac{1}{4} ; \frac{15}{8}\right)$

ordonnée à l'origine : 2

CL41



$\begin{cases} a\cdot 0^2+b\cdot 0+c=0 \Rightarrow c=0 \\ a\cdot 1^2+b\cdot 1+c=1.1 \\ a\cdot 2^2+b\cdot 2+c=1.2 \end{cases}$ $\quad \begin{cases} a+b=1.1 \ (1) \\ 4a+2b=1.2 \ (2) \end{cases}$

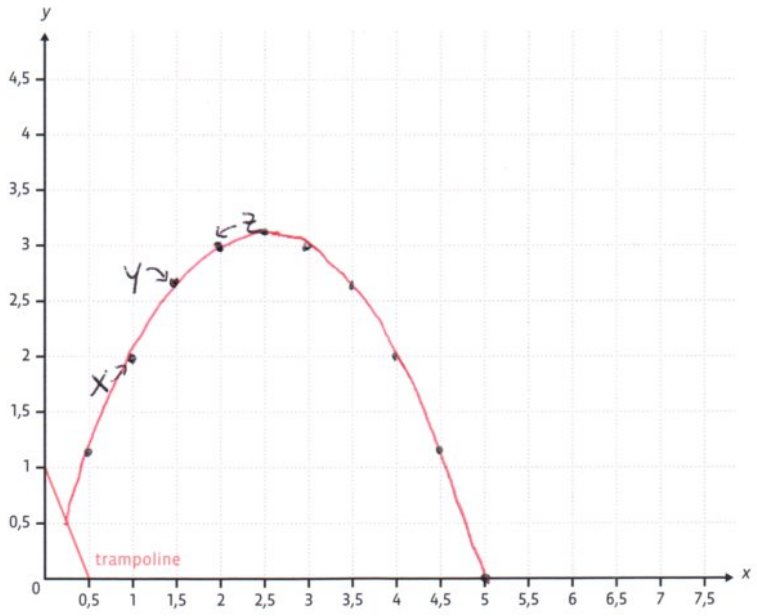
$(1)\cdot 4 \begin{cases} 4a+4b=4.4 \ (3) \\ 4a+2b=1.2 \ (2) \end{cases}$ $\quad (3)-(2)\ 2b=3.2 \Rightarrow b=1.6$

$(1)\ a+1.6=1.1 \Rightarrow a=-0.5$

$f: x \to -0.5x^2+1.6x$

CL42

a.



CL42

b. $\begin{cases} a\cdot 1^2+b\cdot 1+c=2 \\ a\left(\frac{3}{2}\right)^2+b\cdot\frac{3}{2}+c=\frac{21}{8} \\ a\cdot 2^2+b\cdot 2+c=3 \end{cases}$ $\begin{cases} a+b+c=2 & (1) \\ \frac{9}{4}a+\frac{3}{2}b+c=\frac{21}{8} & (2) \\ 4a+2b+c=3 & (3) \end{cases}$

(3)-(1) $\begin{cases} 3a+b=1 & (4) \\ 18a+12b+8c=21 & (5) \\ 32a+16b+8c=24 & (6) \end{cases}$ (2)·8, (3)·8

(6)-(5) $\begin{cases} 3a+b=1 & (4) \\ 14a+4b=3 & (7) \end{cases}$

(4)·4 $\begin{cases} 12a+4b=4 & (8) \\ 14a+4b=3 & (7) \end{cases}$

(8)-(7) : $2a=-1 \Rightarrow a=-\frac{1}{2}$

(4) $-\frac{3}{2}+b=1 \Rightarrow b=1+\frac{3}{2}=\frac{5}{2}$

(1) $-\frac{1}{2}+\frac{5}{2}+c=2 \Rightarrow c=2-\frac{5}{2}+\frac{1}{2}=0$

$$y=\frac{-x^2}{2}+\frac{5}{2}x$$

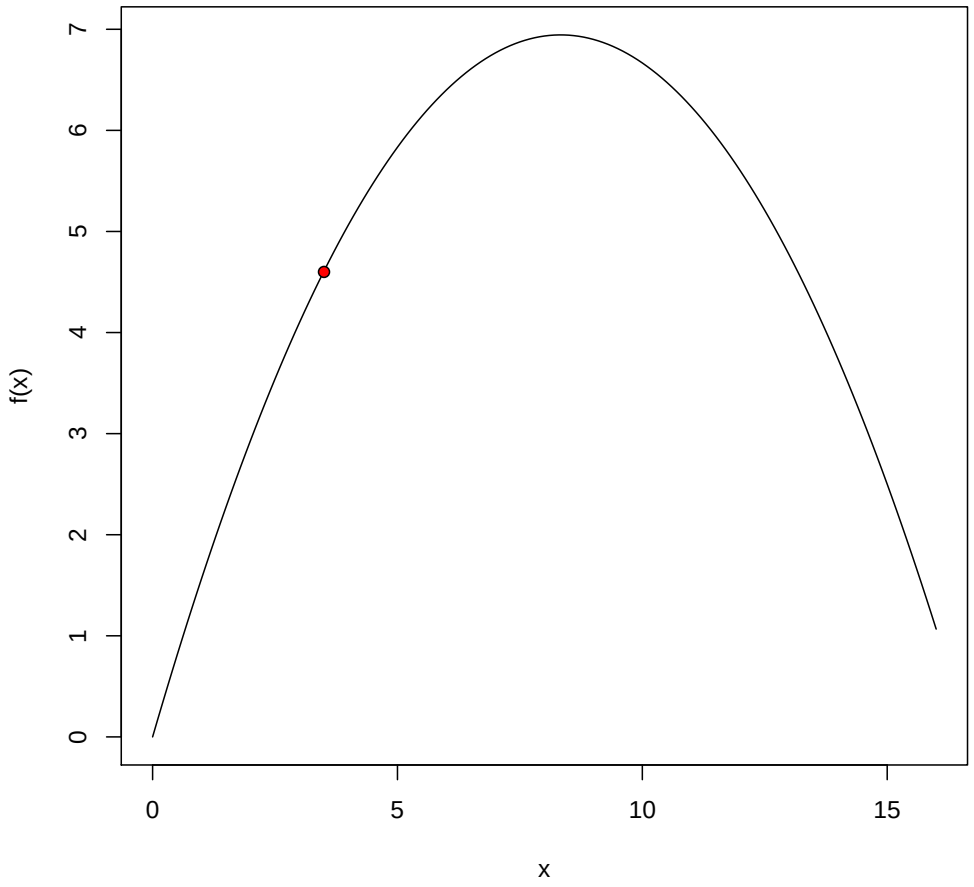
$f(4.5)=\frac{-(4.5^2)}{2}+\frac{5}{2}\cdot 4.5=1,125$ $\Rightarrow$ non le ballon revient un peu plus haut.

c) $\Delta=b^2-4ac=\left(\frac{5}{2}\right)^2-4\left(-\frac{1}{2}\right)\cdot 0=\left(\frac{5}{2}\right)^2=\frac{25}{4}$

$$S=\left(\frac{-\frac{5}{2}}{2\cdot-\frac{1}{2}};\frac{-\frac{25}{4}}{4\cdot-\frac{1}{2}}\right)=\left(-\frac{5}{2}\cdot(-1);\frac{-25}{4}\cdot-\frac{1}{2}\right)$$

$$=\left(\frac{5}{2};\frac{25}{8}\right)=(2.5;3.125)$$

CL 43



b. $f\left(\frac{7}{2}\right) = \frac{-\left(\frac{7}{2}\right)^2}{10} + \frac{5}{3}\cdot\frac{7}{2} = -\frac{49}{40} + \frac{35}{6} = \frac{-294+1400}{240}$

$= \frac{1106}{240} = \frac{553}{120} \simeq 4.608$ m

$\frac{23}{5} = 4.600$ $\Rightarrow$ oui si le boulet possède un rayon d'au moins 8 mm.

c) $f(15.5) = -\frac{15.5^2}{10} + \frac{5}{3}\cdot 15.5 \simeq 1.808$ $\Rightarrow$ oui

CL44

a) la balle est au "sommet" de la parabole

$\Rightarrow -\frac{b}{2a} = 7 \Rightarrow 14a + b = 0$

$(7;-3.6) \in$ parabole $\Rightarrow a \cdot 7^2 + b \cdot 7 + c = -3.6$

$(1;0) \in$ parabole $\Rightarrow a \cdot 1^2 + b \cdot 1 + c = 0$

$$\begin{cases} 14a+b=0 & (1) \\ 49a+7b+c=-3.6 & (2) \\ a+b+c=0 & (3) \end{cases} \quad \begin{matrix} (1) \\ (2)-(3) \end{matrix} \begin{cases} 14a+b=0 \\ 48a+6b=-3.6 \quad (4) \end{cases}$$

$$\begin{matrix} (1)\cdot 6 \\ (4) \end{matrix} \begin{cases} 84a+6b=0 & (5) \\ 48a+6b=-3.6 \end{cases} \quad (5)-(4): 36a = 3.6 \Rightarrow a = 0.1$$

(1) $14 \cdot 0.1 + b = 0 \Rightarrow b = -1.4$

(3) $0.1 - 1.4 + c = 0 \Rightarrow c = 1.3$

$$y = 0.1x^2 - 1.4x + 1.3$$

b) $(5;-2) \in$ parabole : $a5^2 + b \cdot 5 + c = -2$

$$\begin{cases} 14a+b=0 & (1) \\ 49a+7b+c=-3.6 & (2) \\ 25a+5b+c=-2 & (3) \end{cases} \quad \begin{matrix} 2\cdot(1) \\ (2)-(3) \end{matrix} \begin{cases} 28a+2b=0 \quad (4) \\ 24a+2b=-1.6 \quad (5) \end{cases}$$

(4) $-$ (5) : $4a = 1.6 \Rightarrow a = 0.4$

(1) $14 \cdot 0.4 + b = 0 \Rightarrow b = -5.6$

(3) $25 \cdot 0.4 + 5 \cdot (-5.6) + c = -2$

$10 - 28 + c = -2$

$c = 16$

$$y = 0.4x^2 - 5.6x + 16$$

CL46

a) $m \cdot 1^2 + 2 \cdot 1 + m = 2 \Rightarrow 2m + 2 = 2 \Rightarrow m = 0$

$x \mapsto 2x$

b) $m \cdot 5^2 + m \cdot 5 - 2 = 7 \Rightarrow 25m + 5m - 2 = 7$

$30m = 9 \Rightarrow m = \frac{9}{30} = \frac{3}{10}$

$\Rightarrow x \mapsto \frac{3}{10}x^2 + \frac{3}{10}x - 2$

c) $m \cdot 1^2 + m^2 \cdot 1 - 2 = 0$; $m^2 + m - 2 = 0$

$(m+2)(m-1) = 0$; $m = -2$ ou 1

$f_{-2} : x \mapsto -2x^2 + 4x - 2$

$f_1 : x \mapsto x^2 + x - 2$

d) $2(-3)^2 + m \cdot (-3) - 2(m+1) = 8$

$18 - 3m - 2m - 2 = 8$; $-5m = -8$; $m = \frac{8}{5}$

$x \mapsto 2x^2 + \frac{8}{5}x - \frac{26}{5}$

e) $2m(-2)^2 + (m-3)(-2) - 6m = 6$

$8m - 2m + 6 - 6m = 6$

$8m - 8m = 0$; $0 = 0$

$\Rightarrow$ toutes les fonctions de la famille $x \mapsto 2mx^2 + (m-3)x - 6m$ passent par $(-2;6)$, quelque soit la valeur de m.

f) $2^2-(m+2)(m-2)+10m+6=-10$

$4-(m^2-2m+2m-4)+10m+6=-10$

$4-m^2+4+10m+6=-10$

$-m^2+10m+14=-10$

$-m^2+10m+24=0 \quad |\cdot -1$

$m^2-10m-24=0$

$(m-12)(m+2)=0 \Rightarrow m=-2$ ou 12

si $m=-2$: $x^2-(-2+2)(-2-x)+10(-2)+6$

$=x^2-0(-2-x)-14$

$\Rightarrow f_{-2}: x \longmapsto x^2-14$

si $m=12$: $x^2-(12+2)(12-x)+126=$

$x^2-14(12-x)+126=x^2-168+14x+126$

$\Rightarrow f_{12}: x \longmapsto x^2+14x-42$

g) $-3^2+(2m-3)\cdot 3-6m=-9$

$-9+6m-9+6m=9$

$-18=9$

$\Rightarrow$ Quelque soit la valeur de m, aucune fonction de la famille $x \longmapsto x^2+(2m-3)x-6m$ ne passe par $(3;9)$.

CL47

a) $\begin{cases} a\cdot 1^2+b\cdot 1+c=m+4 \\ a\cdot 2^2+b\cdot 2+c=2m+10 \\ a\cdot 0^2+b\cdot 0+c=6 \Rightarrow c=6 \end{cases}$ $\begin{cases} a+b+6=m+4 \quad (1) \\ 4a+2b+6=2m+10 \quad (2) \end{cases}$

$(1)\cdot 2$ $\begin{cases} 2a+2b+12=2m+8 \\ (2) \quad 4a+2b+6=2m+10 \end{cases}$ $(2)-(1)$: $2a-6=2$

$\Rightarrow 2a=8 \Rightarrow a=4$

(1) $4+b+6=m+4$; $b+10=m+4 \Rightarrow b=m-6$

$p: x \longmapsto 4x^2+(m-6)x+6$

b) 1. $\begin{cases} a\cdot 1^2 + b\cdot 1 + c = m+1 \\ a\cdot 0^2 + b\cdot 0 + c = 1 \Rightarrow c = 1 \\ a\cdot 2^2 + b\cdot 2 + c = m+4 \end{cases}$

$\begin{cases} a+b+1 = m+1 \\ 4a+2b+1 = m+4 \end{cases}$ $\quad\quad$ $\begin{cases} a+b = m & (1) \\ 4a+2b = m+3 & (2) \end{cases}$

(1)·2 $\begin{cases} 2a+2b = 2m \\ (2)\ 4a+2b = m+3 \end{cases}$ $\quad$ (1)−(2) $-2a = m-3$

$\Rightarrow a = \dfrac{-(m-3)}{2} = \dfrac{3-m}{2}$

(1) $\dfrac{3-m}{2} + b = m$

$b = m - \dfrac{3-m}{2}$

$b = \dfrac{2m-(3-m)}{2} = \dfrac{3m-3}{2} = \dfrac{3(m-1)}{2}$

$p : x \mapsto \dfrac{3-m}{2}x^2 + \dfrac{3(m-1)}{2}x + 1$

2 droite AB : $\begin{cases} m+1 = a\cdot 1 + b \\ 1 = a\cdot 0 + b \Rightarrow b = 1 \end{cases}$

$m+1 = a+1 \Rightarrow a = m \Rightarrow y = mx+1$

$C \in AB \Rightarrow m+4 = m\cdot 2 + 1 = 2m+1$

$\Rightarrow m = 3$

A, B et C sont alignés si $m = 3$

CL 49

Droite tangente à la parabole ⇒ un seul point d'intersection ⇒ Δ de la parabole $= 0$

a) $mx + 5 = -x^2 + 3x + 4$

$-x^2 + (3-m)x - 1 = 0 \quad | \cdot (-1)$

$x^2 + (m-3)x + 1 = 0$

$\Delta = (m-3)^2 - 4 \cdot 1 \cdot 1 = (m-3)\cdot(m-3) - 4 = m^2 - 6m + 5$

$= (m-1)(m-5) = 0$

$\Rightarrow m \in \{1; 5\}$

Si $m = 1$:

$x_T^2 - 2x + 1 = 0$

$(x_T - 1)^2 = 0 \Rightarrow x_T = 1$

$y_T = 1 \cdot 1 + 5 = 6 \qquad T(1; 6)$

Si $m = 5$:

$x_T^2 + 2x + 1 = 0$

$(x_T + 1)^2 = 0 \Rightarrow x_T = -1$

$y_T = 5 \cdot (-1) + 5 = 0 \qquad T(-1; 0)$

b) $4x + m = 2x^2 + mx - 4$

$2x^2 + (m-4)x - (m+4) = 0$

$\Delta = (m-4)^2 - 4 \cdot 2 \cdot -(m+4) = (m-4)^2 + 8(m+4) =$

$m^2 - 8m + 16 + 8m + 32 = m^2 + 48$

$\Delta = 0 \Rightarrow m^2 = -48 \Rightarrow$ Jamais tangente, toujours sécante

c) $4x + m = 2x^2 + mx + 4$

$2x^2 + mx - 4x - m + 4 = 0$

$2x^2 + (m-4)x - (m+4) = 0$

$2x^2 + (m-4)x + (4-m) = 0$

$\Delta = (m-4)^2 - 4 \cdot 2 \cdot (4-m) = m^2 - 8m + 16 - 32 + 8m =$

$m^2 - 16 = (m-4)(m+4) = 0$

$\Rightarrow m \in \{-4; 4\}$

si $m = -4$:

$2x_T^2 + (-4-4)x_T + (4-(-4)) = 2x_T^2 - 8x_T + 16 =$

$2(x_T^2 - 4x_T + 4) = 2(x_T - 2)^2 = 0 \Rightarrow x_T = 2$

$y_T = 4 \cdot 2 + (-4) = 4 \Rightarrow \underline{\underline{T(2;4)}}$

si $m = 4$:

$2x_T^2 + (4-4)x + (4-4) = 2x_T^2 = 0 \Rightarrow x_T = 0$

$y_T = 4 \cdot 0 + 4 = 4 \quad \underline{\underline{T(0;4)}}$

CL53

axe de symétrie : $x = \frac{-b}{2a}$

Sommet : $\left(-\frac{b}{2a}; -\frac{b^2-4ac}{4a}\right)$

a) $\frac{-b}{2a} = 0 \Rightarrow b = 0 \Rightarrow \underline{\underline{y = ax^2 + c}}$

b) $y = ax^2 + c$

$(0;4) \in$ parabole $\Rightarrow 4 = a0^2 + c \Rightarrow c = 4$

$\Rightarrow \underline{\underline{y = ax^2 + 4}}$

c) $y = ax^2 + c$

$(2;5) \in$ parabole $\Rightarrow 5 = a \cdot 2^2 + c$

$\Rightarrow c = 5 - 4a \Rightarrow \underline{\underline{y = ax^2 - 4a + 5}}$

d) axe de symétrie : $x = -0.5 = \frac{-b}{2a} \Rightarrow b = a$ (1)

$(2;0) \in$ parabole $\Rightarrow$

$0 = a \cdot 2^2 + b \cdot 2 + c$ (2)

(1) dans (2) $4a + 2a + c = 0 \Rightarrow c = -6a$

$\Rightarrow \underline{\underline{y = ax^2 + ax - 6a}}$

e) axe de symétrie : $x = -3 = \frac{-b}{2a} \Rightarrow b = 6a$

$(0;0) \in$ parabole $\Rightarrow 0 = a \cdot 0^2 + 6 \cdot 0 + c \Rightarrow c = 0$

$\Rightarrow \underline{\underline{y = ax^2 + 6ax}}$

f) sommet $(4;0) \Rightarrow \frac{-b}{2a} = 4 \Rightarrow b = -8a$

et $-\frac{b^2 - 4ac}{4a} = 0 \Rightarrow \frac{(-8a)^2 - 4ac}{4a} = 0$

$\frac{64a^2 - 4ac}{4a} = 0$

$\frac{4a(16a - c)}{4a} = 0 \quad a \neq 0$

$16a - c = 0 \Rightarrow c = 16a \Rightarrow \underline{\underline{y = ax^2 - 8ax + 16a}}$

g) $\frac{-b}{2a} = -2 \Rightarrow b = 4a \Rightarrow \underline{\underline{y = ax^2 + 4ax + c}}$

h) $\frac{-b}{2a} = 1 \Rightarrow b = -2a$

$(0;-2) \in$ parabole $\Rightarrow -2 = a0^2 + (-2a) \cdot 0 + c$

$\Rightarrow c = -2 \Rightarrow \underline{\underline{y = ax^2 - 2ax - 2}}$

i) $\frac{-b}{2a} = 3 \Rightarrow b = -6a$

$-\frac{b^2-4ac}{4a} = 5 \Rightarrow \frac{36a^2-4ac}{4a} = -5$

$\frac{4a(9a-c)}{4a} = -5 \; ; \; a \neq 0$

$9a - c = -5 \Rightarrow c = 9a + 5 \Rightarrow \underline{\underline{y = ax^2 - 6ax + 9a + 5}}$

j) $\frac{-b}{2a} = -4 \Rightarrow b = 8a$

$-\frac{(8a)^2-4ac}{4a} = -7$

$\frac{64a^2-4ac}{4a} = 7$

$64a^2 - 4ac = 28a$

$64a^2 - 4ac - 28a = 0$

$c = \frac{64a^2-28a}{4a} = \frac{4a(16a-7)}{4a} \quad a \neq 0$

$c = 16a - 7 \Rightarrow \underline{\underline{y = ax^2 + 8ax + 16a - 7}}$

CL 64

a) $x^2 + 6x + 8 = (x+2)(x+4)$

x		-4		-2	
$x+2$	$-$		$-$	0	$+$
$x+4$	$-$	0	$+$		$+$
x^2+6x+8	$+$	0	$-$	0	$+$

b) $2x^2-2x-24 = 2(x^2-x-12) = 2(x+3)(x-4)$

x		-3		4	
$x+3$	$-$	0	$+$		$+$
$x-4$	$-$		$-$	0	$+$
$2x^2-2x-24$	$+$	0	$-$	0	$+$

c) $9x^2-x-10 = 9(x+1)\left(x-\frac{10}{9}\right)$

x		-1		$\frac{10}{9}$	
$x+1$	$-$	0	$+$		$+$
$x-\frac{10}{9}$	$-$		$-$	0	
$9x^2-x-10$	$+$	0	$-$	0	$+$

d) $-x^2-x+42 = -(x-6)(x+7)$

x		-7		6	
$-(x-6)$	$+$		$+$	0	$-$
$(x+7)$	$-$	0	$+$		$+$
$-x^2-x+42$	$-$	0	$+$	0	$-$

e) $4x^2-3x-27 = \left(x+\frac{9}{4}\right)(x-3)$

x		$-\frac{9}{4}$		3	
$x+\frac{9}{4}$	$-$	0	$+$		$+$
$x-3$	$-$		$-$	0	$+$
$4x^2-3x-27$	$+$	0	$-$	0	$+$

f) $x^3 - 2x^2 - 15x = x(x^2 - 2x - 15) = x(x+3)(x-5)$

x		-3		0		5	
x	$-$		$-$	0	$+$		$+$
$x+3$	$-$	0	$+$		$+$		$+$
$x-5$	$-$		$-$		$-$	0	$+$
x^3-2x^2-15x	$-$	0	$+$	0	$-$	0	$+$

g) $x \to x^4 - x^2 = x^2(x^2-1) = x^2(x-1)(x+1)$

x		-1		0		1	
x^2	$+$		$+$	0	$+$		$+$
x^2-1	$+$	0	$-$		$-$	0	$+$
x^4-x^2	$+$	0	$-$	0	$-$	0	$+$